Newton & Stokes, Laminar vs. Turbulent Flow

Joseph G. Theiss
Basement, Physics Department, The College of Wooster, Wooster, Ohio 44691, USA
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It was found that Stokes' model for drag fit only on smooth objects or non-smooth objects at low speeds. The standard deviation from the predicted values was an average of 1.10e-05 in the linear looking ranges. Newton's model was also fit to linear ranges and had deviations of an average 4.49e-06. Unlike Stokes' model, Newton's predicted the movement of fast non-smooth objects. Objects had transition periods where they would go from Newton's model to Stokes' as the objects would slow down. This transition period was largely ignored and should be investigated in further experiments.

INTRODUCTION

Fluid dynamics is a large field encompassing how a fluid reacts on a surface or with itself in almost all circumstances. The drag force is a force that can occur when a fluid is moved over a surface. This broad definition can be applied to many situations, from windmills to NASCAR cars to planes. In all of these examples the drag force can make or brake a design for the respective industry. NASCAR cars want to minimize drag to increase the maximum speed of a car. A plane and a windmill want different amounts of drag on each side of their control surfaces to facilitate movement.

There are two kinds of drag which can occur when a fluid moves over a surface. Laminar and Turbulent Flow. These are now two very distinct kinds of drag, but in Newton's and Stokes' time this was not well defined. Two prominent figures in science - Stokes and Newton - argued for there own model. Little did they know they were both correct, but in different circumstances.

This experiment goes back to this argumentative time and shows how different surfaces can cause these different models and also how speed affects them too. We will use a large metal ball, with negligible friction, as our surface with mountable flags. The spun ball will interact with our fluid, the air, and cause drag on the surface.

THEORY

An object can accelerate with a certain angular acceleration α - in our case deceleration - with a change in angular velocity ω per time t as shown in

$$\alpha = \frac{\Delta\omega}{\Delta t}. \tag{1}$$

This acceleration is the result from some force acting on the outside surface of the sphere r, our drag force F_d, causing a braking torque τ. This is the only force acting on the angular acceleration as the ball is floating on a pad of gas, and so it is the only torque as well. Using Fig. 1 we can write and resolve the torque as

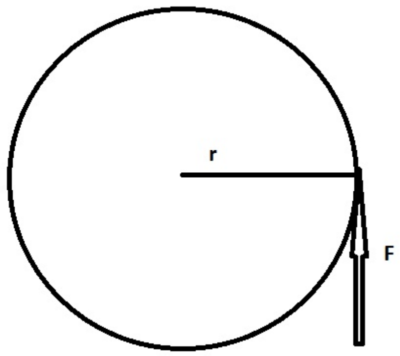

FIG. 1: How the vector $\vec{r}$ is related in space to the vector $\vec{F_d}$

$$\tau \equiv \vec{r} \times \vec{F_d}, = rF_d. \tag{2}$$

Torque is also defined to be

$$\tau \equiv I\alpha \tag{3}$$

where I is the moment of inertia of the spinning object. By setting Eq. (2) and Eq. (3) equal to each other, we can solve for F_d as

$$F_d = \frac{I\alpha}{r}. \tag{4}$$

By further substituting Eq. (1) into Eq. (4) we get the relationship

$$F_d = \frac{I}{r}\frac{\Delta\omega}{\Delta t} \tag{5}$$

$$F_d = \frac{I}{r}\frac{d\omega}{dt} \tag{6}$$

Newton and Stokes argued against each other on how the drag force was affected by the velocity of the surface v

it was acting on. Newton argued that this drag force was proportional to the square of velocity and Stokes argued it is a linear relationship. These two view points can be represented as

$$F_d \propto v^n \tag{7}$$
$$F_d \propto (2\pi r\omega)^n \tag{8}$$
$$F_d \propto \omega^n \tag{9}$$

Where n is the place holder value for either 1 or 2. We can use these two equations - Eq. (6) & Eq. (9) - to get the differential equation

$$\omega^n \propto \frac{d\omega}{dt} \tag{10}$$
$$dt \propto \frac{d\omega}{\omega^n}. \tag{11}$$

Stokes' case, n = 1 Laminar Flow

Stokes, when doing his drag experiments in the 1800's, preformed his experiment in exclusively narrow tubes. These narrow tubes caused what we now call Laminar Flow, that is the flow's velocity remains the same within a region. This is shown in Fig. 2.

Solving the differential equation - Eq. (11) - using Stokes' case for the drag force shows.

$$\frac{d\omega}{\omega} \propto dt \tag{12}$$
$$\ln(\omega) - \ln(\omega_o) = kt \tag{13}$$
$$\ln(\omega) = \ln(\omega_o) + kt. \tag{14}$$

According to the above equation, graphing $\ln(\omega)$ vs. t should give a straight line with a slope of some proportionality k and an intercept of ω_o

Newton's case, n = 2 Viscous Flow

Newton, in his Principia Mathematica[1], envisioned fluid flow as turbulent. Where the magnitude and direction of the velocity vectors of the fluid were mismatched and seemingly random as shown in Fig. 2.

Solving the differential equation - Eq. (11) - using Newton's case for the drag force shows.

$$\frac{d\omega}{\omega^2} \propto dt \tag{15}$$
$$\frac{1}{\omega_o} - \frac{1}{\omega} = kt \tag{16}$$
$$\frac{1}{\omega} \propto \frac{1}{\omega_o} - kt \tag{17}$$

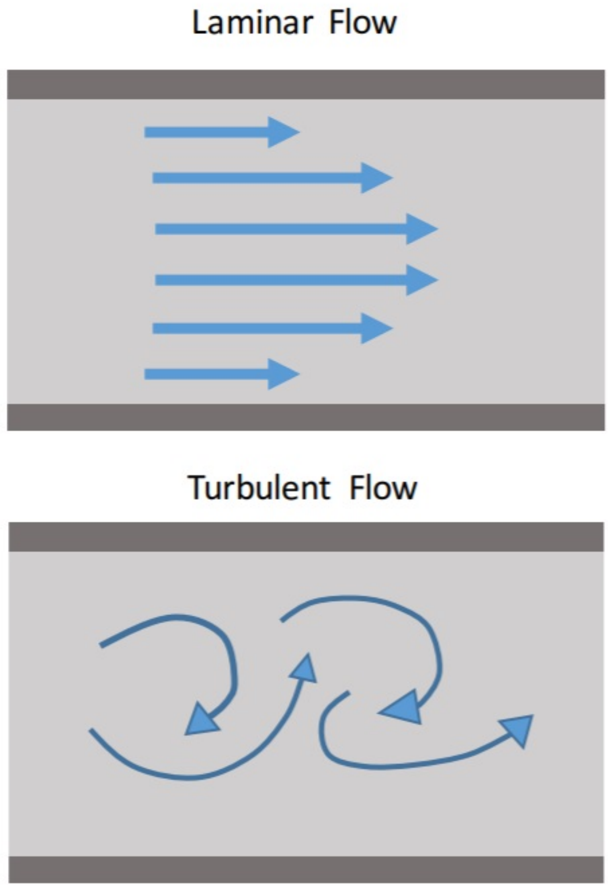

FIG. 2: The vector fields of a flow for the two models. The turbulent case was predicted by Newton and has random vector directions. The other case was predicted by Stokes and is the Laminar flow.

According to the above equation, graphing $\frac{1}{\omega}$ vs. t should give a straight line with a slope of some proportionality k and an intercept of $\frac{1}{\omega_o}$

PROCEDURE

Shown in Fig. 3 is the setup used in this experiment. It shows how a He-Ne laser reflects off a sphere with interrupting non-reflective strips. These non-reflective strips cause the reflected laser light to become periodic when the ball is rotating. This periodic motion is picked up by photo-diode through a focusing lens. The signal from the photo-diode is put through a Schmidt trigger to a frequency counter as shown in Fig. 4.

The counted frequency is recorded on a nearby computer running Lab-view in $10s$ intervals. This saves the average frequency in the interval with an attached time. These data are used to calculate two different least square fits - according to Stokes and Newton - and the standard deviation from the lines.

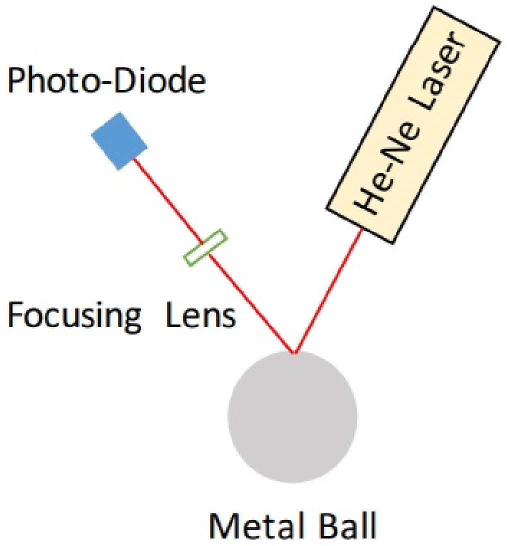

FIG. 3: The setup used in this experiment consisted of a He-Ne laser which was periodically reflected of a spinning ball and sent to the photo-diode

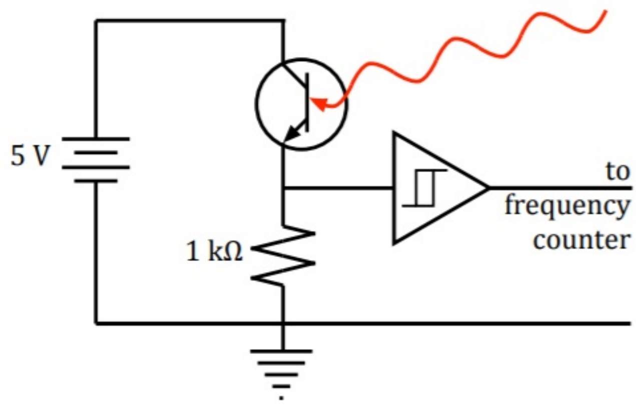

FIG. 4: The Schmidt trigger takes the signal from the photo-diode, pulses when the photo-diode is triggered, and sends the signal to the frequency counter. Sorced from Dr. Lehman at the College of Wooster [2].

Four tests were ran to see a wide range of scenarios. The metal sphere has a long peg attached to it from which a flag can be tapped to. The flag used in the experiments was an 11cm x 8cm piece of A4 paper. The tests scenarios, represented in Fig. 5 were

- No Flag
- Bottom - The flag was centered in the middle of the peg at the bottom
- Top - The flag was centered in the middle of the peg at the top
- Side - The flag was placed at the side of the peg in the middle

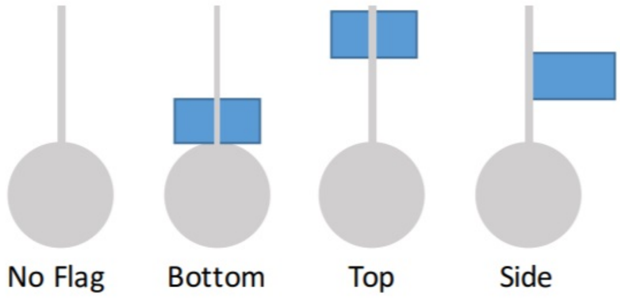

FIG. 5: A visual of the flag with its position on the metal ball's peg.

RESULTS & ANALYSIS

Looking at Fig. 6 & Fig. 7 we can see that there are ranges in both where the behavior is linear. There is no definitive transition however between this linearity and the non-linearity. These ranges of transitions are not well defined too and can take different amounts of time to reach and go through based on the object in the fluid. These periods of switching is where the flow over the object is between Laminar flow and Turbulent flow.

For example, the Side position reached its linearity in Fig. 6 in under 200 seconds, but the same flag at the bottom or top position takes well over 200 seconds to reach its linear range. This is also shown in Fig. 7, as it takes roughly the same amount of time to leave its linear range in that figure.

This switching between non-linear and linear ranges are caused from which model, either Stokes' or Newton's, is correctly predicting motion. These modes can switch based on the speed of the ball.

Laminar Flow, Stokes' Model

Fig. 6 uses Stokes' Model where linear ranges represent the where the model is correctly predicting changes in ω from the drag force. This model takes over consistently on smooth objects which makes sense as the fluid doesn't curl around a surface and its flow is parallel to the interacting surface. This can also occur at slower velocities as shown by the figure. In this case the fluid travels so slow there is little curl around the non-smooth surfaces. The values of the standard deviation for these ranges are given in Table I. The larger the deviation the worse the predictive power of the theory for the range measured.

Turbulent Flow, Newton's Model

Fig. 7 uses Newton's Model where linear ranges represent the where the model is correctly predicting changes

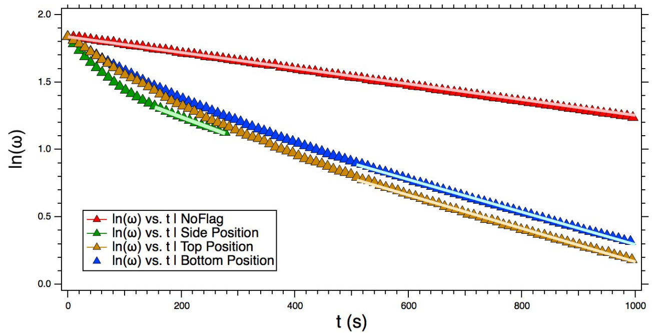

FIG. 6: A fit was done on the idea of Eq. (14). This fit, when linear, represents a fit to his theory. We can see a dynamic range where his theory fits and another where it does not.

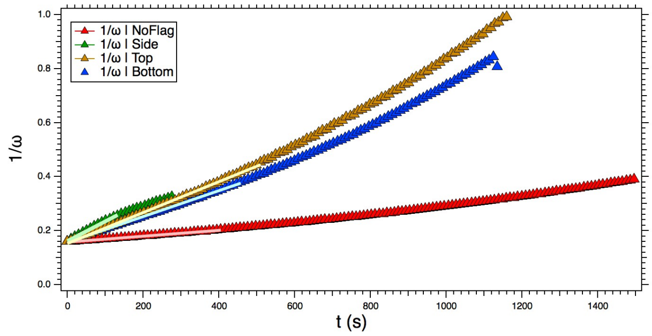

FIG. 7: A fit was done on the idea of Eq. (17). This fit, when linear, represents a fit to his theory. We can see a dynamic range where his theory fits and another where it does not.

in ω from the drag force. You can see this model predicting change correctly during high speed applications - early on in time - on non-smooth surfaces - those with flags. This is important to realize as much of the world is non-smooth and has angular shapes; so then this model would be needed to help calculate drag on these kind of objects. The values of the standard deviation for these ranges are given in Table II. The larger the deviation the worse the predictive power of the theory for the range measured.

TABLE I: This shows the Standard deviation from the least squares fit on Fig. 6. The smaller the number, the better the fit, and the better the predictive power of the theory

Position	Standard Deviation
No Flag	3.13e-07
Bottom	5.39e-06
Top	8.15e-06
Side	3.04e-05

TABLE II: This shows the Standard deviation from the least squares fit on Fig. 7. The smaller the number, the better the fit, and the better the predictive power of the theory.

Position	Standard Deviation
No Flag	6.84e-07
Bottom	1.78e-06
Top	2.93e-6
Side	1.26e-05

CONCLUSION

Both models are necessary to calculate drag in different circumstances. It was found that in low speed applications that Stokes' Model of Laminar Flow would correctly predict changes in speed from the drag force. At high speed applications with non-smooth surfaces Newton's Model of Turbulent Flow had a strong predictive power. While this experiment was easily able to show how the two models could dominate a certain scenario it did not however handle the transition between these models. There are parts of each non-smooth object's movement where neither Stokes' or Newton's Model could handle it well. This should be an area of further research.

[1] Isaac Newton, *Philosophi Naturalis Principia Mathematica* (1687).
[2] Susan Lehman, *Jr IS Lab Manual* (College of Wooster Physics Department, SPRING 2018).